Table 1 Comparison of calculated and experimental values of cratering energy density

Target	Projectile Material	Projectile Density ρ, g/cm³	Particle velocity v_p	E from Eq (8), ergs/cm³	E from experiment, ergs/cm³	Reference
Al-1100	SiC	3 2	8,400 fps	$6\ 26 \times 10^9$	$2\ 36 \times 10^{10}$	Eq (6)
Al-2014	WC	15 6	<49,200 fps	$1\ 28 \times 10^9$	$2\ 25 \times 10^{10}$	Ref 3
Al-75 St	Steel	7 83	<49,200 fps	$2\ 56 \times 10^9$	$3\ 90 \times 10^{10}$	Ref 3
Al-2024-T3 and 24 St	Al	2 7	8,000 fps	$7\ 42 \times 10^9$	$2\ 49 \times 10^{10}$	Ref 4

the cratering energy density, that is, the projectile energy required per unit volume of the crater formed in the target

Over the range of impact energies of interest here, the rate of surface damage can be assumed proportional to remaining undamaged area Thus

$$dA_D/d\epsilon = K_1[A_0 - A_D(\epsilon)] \tag{2}$$

where

$A_D(\epsilon)$ = area damaged by exposure to ϵ
A_0 = area of undamaged $\frac{15}{16}$ in -diam disk

The simplest procedure for relating K_1 and E_c is to consider the initial condition of an undamaged surface Then, for the first hit $A_D(0) = 0$, and Eq (2) can be written

$$(dA_D/d\epsilon)_{\epsilon\approx 0} = K_1 A_0 \tag{3}$$

If the crater volume is proportional to the projectile energy, the left side can also be written

$$\left(\frac{dA_D}{d\epsilon}\right)_{\epsilon\approx 0} = \frac{(3\pi^{1/2} m_p v_p{}^2/4E_c)^{2/3}}{(\frac{1}{2}m_p v_p{}^2)} \tag{4}$$

where the numerator is the damaged area due to the first hit, and the denominator is the kinetic energy of first particle impingement, here in ergs From Eqs (3) and (4)

$$K_1 = \frac{2}{A_0}\left(\frac{3\pi^{1/2}}{4E}\right)^{2/3} \frac{1}{(m_p v_p{}^2)^{1/3}} \tag{5}$$

or

$$E_r = \frac{3\pi^{1/2}}{2^{1/2}(K_1 A_0)^{3/2}(m_p v_p{}^2)^{1/2}} \tag{6}$$

For our experimental condition (with v_p = 8400 fps) the value for E obtained from Eq (6) is given in Table 1 for an Al target Table 1 also contains the experimental value for E_c from Refs 3 and 4, as determined by impacting high-speed projectiles against Al targets and measuring the volume of the crater formed in the targets It can be seen in Table 1 that the value for E_c, as determined from Eq (6), compares well with the experimental values of Refs 3 and 4 It should be pointed out that the value for K_1 necessary for this calculation was obtained from only six reflectivity measurements These reflectivity measurements were quite good, but further experimentation would provide a more accurate value for K_1

In Ref 5, the following equation is presented for the penetration of high-speed projectiles into semi-infinite targets:

$$p/D = 2\ 28(\rho_p/\rho_T)^{2/3}(v_p/C)^{2/3} \tag{7}$$

where

p = penetration
D = diameter of particle
ρ_p = projectile density
ρ_T = target density
v_p = particle velocity
C = speed of sound in target

This equation has been extensively used to determine the number of penetrations expected to occur in satellite penetration experiments [6] From this equation, E_c is

$$E = (C^2/94\ 4)(\rho_T{}^2/\rho_p) \tag{8}$$

Values for E_c calculated from Eq (8) are also presented in Table 1 for comparison with the experimentally determined values Notice that Eq (8) underestimates, considerably, the cratering energy density as compared with the experiments and, hence, would overestimate penetrations expected to occur in Al This suggests that great care must be taken when trying to determine meteoroid flux from satellite penetration experiments, and that flux estimates from these experiments may be low and are very approximate at best The possibility of measuring micrometeoroid flux in the vicinity of the earth with a calibrated reflectivity sensor is being considered

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Calorimetric Heating-Rate Probe for Maximum-Response-Time Interval

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Nomenclature

c = slug specific heat
k = thermal conductivity
q = heating rate
t = temperature
t_m = maximum allowable front-face temperature
x = length dimension
α = thermal diffusivity
δ = length of slug
ρ = slug density
θ = time

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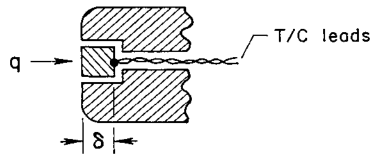

Fig 1 Aerodynamic heating-rate probe

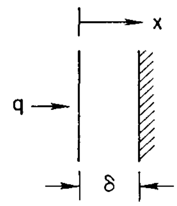

Fig 2 One-dimensional model

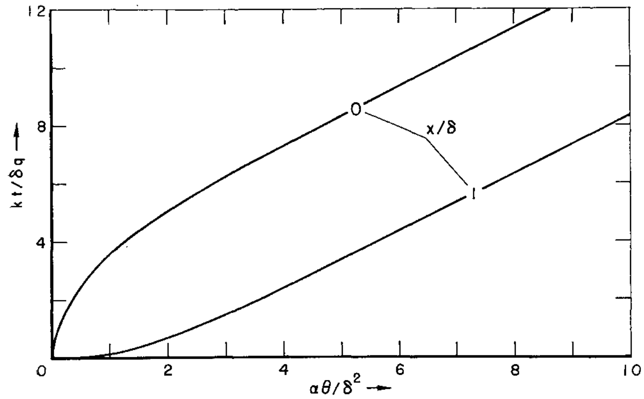

Fig 3 Temperature response of a plate

A DESIRABLE feature of calorimetric probes for very high heating-rate environments is the longest possible interval of linear temperature-time response It is the purpose of this note to indicate a design for a calorimetric probe that will maximize the time interval of linear response

A typical aerodynamic heating-rate probe is shown in Fig 1 The frontal area of the calorimetric slug is small enough that only stagnation-point heat transfer is measured Consider the probe to be a finite slab of material with a uniform heating rate at the front face and to be insulated on the sides and at the back face, thus reducing the problem to one-dimensional heat conduction as shown in Fig 2

The solution of the one-dimensional heat-conduction equation

$$\partial^2 t/\partial x^2 = (1/\alpha)(\partial t/\partial \theta) \qquad (1)$$

subject to the boundary conditions

$$\begin{array}{ll} x = 0 & q = \text{const} \\ x = \delta & \partial t/\partial x = 0 \\ \theta = 0 & t = 0 \end{array}$$

gives the temperature distribution in the model of Fig 2 The solution is given by Carslaw and Jaeger [1] The variation of temperature with time is plotted by Schneider,[2] as shown in Fig 3

From Fig 3, and for a given material, a value of θ_1, the time required for the back face of the slug to reach a state of linear temperature increase with time can be established The time for the back face to undergo its initial transient may be taken from Fig 3 as

$$\theta_1 = 0\ 35\ \delta^2/\alpha \qquad (2)$$

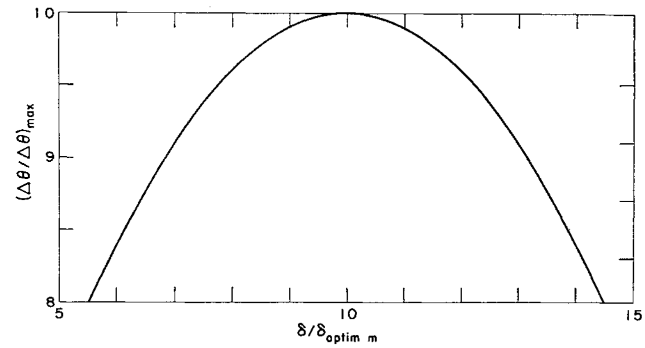

Fig 4 Plot of Eq (7)

From the Carslaw and Jaeger solution, the front-face temperature after the initial transient is also a linear function of time and is given as

$$t = \delta q/k[(\alpha\theta/\delta^2) + \tfrac{1}{3}] \qquad (3)$$

The front face reaches a maximum allowable temperature t_m in time θ_2 Equation (3) then yields

$$\theta_2 = \delta^2/\alpha[(kt_m/\delta q) - \tfrac{1}{3}]$$

In deducing heating rate from the temperature-time response curve, the following equation is employed:

$$q = \rho\delta c(\Delta t/\Delta\theta) \qquad (4)$$

For extremely high heating rates, the time interval $\Delta\theta = \theta_2 - \theta_1$ (the linear portion of the temperature-time response of the attached thermocouple) can be very small, and thus difficult to measure In optimizing a probe design, it is therefore desirable to have $\theta_2 - \theta_1$ a maximum

Combining Eqs (2) and (3), one has

$$\Delta\theta = (kt_m/\alpha q)\delta - (0\ 683\ \delta^2/\alpha) \qquad (5)$$

The quantity $\Delta\theta$ now may be maximized as a function of the thickness δ:

$$d(\Delta\theta)/d\delta = (kt_m/\alpha q) - [2(0\ 683)\delta/\alpha] = 0$$

Therefore, the optimum value of thickness becomes

$$\delta_{\text{opt}} = kt_m/1\ 366\ q \qquad (6)$$

After substitution of Eq (6) into Eq (5), the maximum time interval is found to be

$$\Delta\theta_{\max} = 0\ 366(k^2 t_m{}^2/\alpha q^2) \qquad (7)$$

For convenience, Eq (5) may be normalized to yield

$$\Delta\theta/\Delta\theta_{\max} = (2\delta/\delta_{\ \text{pt}}) - (\delta/\delta_{\text{opt}})^2 \qquad (8)$$

Equation (8) is plotted in Fig 4

The calorimetric probe designed by means of Eq (6) will give the longest possible linear temperature-time response for a given material, allowable front-face temperature rise, and fixed heating rate

References

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